) A non-recursive, non-iterative definition of transpose

transpose $\overset{df}{=} (p \rightarrow f ; \delta b) \cdot g \cdot h$

where p. 2

$p = (\text{insert and}) \cdot \alpha\,\text{null} \cdot 1$

$f = \alpha\alpha 1 \cdot tl$

$g = \alpha(\text{power } \alpha tl)$

$h = \text{rotr} \cdot \text{distr} \cdot [\, r, id \,]$

$r = jn \cdot [(\text{constant } 0), \text{ iota} \cdot ln \cdot 1\,]$ ← see Feb 14 notes, p. 8

and $\delta b : x = \bot$, all x

Descriptions

(a) $r : x = \langle 0, 1, \ldots, m \rangle$ when $ln : x \geq 1$ and $ln \cdot 1 : x = m \geq 0$

(b) $h : x = \langle \langle m, x \rangle, \langle 0, x \rangle, \ldots \langle m-1, x \rangle \rangle$
when $ln : x \geq 1$ and $m \geq 1$ $\quad m = ln \cdot 1 : x$
$= \langle\langle 0\ x \rangle\rangle$ when $m = 0$

(c) $g : h : x = \langle (\alpha tl)^m : x,\ x,\ \alpha tl : x, \ldots, (\alpha tl)^{m-1} : x \rangle$

(d) $(\alpha tl)^k : x \equiv \langle tl^k : x_1, \ldots, tl^k : x_n \rangle$ where $x = \langle x_1, \ldots, x_n \rangle$
thus $(\alpha tl)^m : x = \langle \phi, .\phi., \phi \rangle$ iff $ln : x_i =$ ~~ln:x~~ $m \quad i = 1, \ldots, n$
thus $p : g : h : x = T$ iff $ln : x_i = m$

(e) $f : g : h : x = \langle \alpha 1 : x,\ \alpha 1 : \alpha tl : x, \ldots, \alpha 1 : (\alpha tl)^{m-1} : x \rangle$

(f) $\alpha 1 : (\alpha tl)^k : x = \langle (k+1) : x_1, \ldots, (k+1) : x_n \rangle$
$= \langle x_{1,k+1}, \ldots, x_{n,k+1} \rangle$
where $x_i = \langle x_{i,1}, \ldots, x_{i,m} \rangle$

thus
$f : g : h : x = \langle y_1, \ldots, y_m \rangle$
where $y_i = \langle x_{1,i}, \ldots, x_{n,i} \rangle$
when $p : g : h : x = T$

(i) An iterative definition of transpose

$$\text{transpose} \stackrel{df}{=} f_1 \cdot (\text{while } p\ f_2) \cdot f_3$$

where

$$f_3 = [(\text{constant } \emptyset), \overset{id}{\cancel{\alpha\,\text{rotr}}}]$$

$$f_2 = [r, \alpha \overset{tail}{\cancel{(\text{rotr} \cdot \text{tl})}} \cdot 2]$$ rotr:$\emptyset$ illegal !

$$r = \text{rotl} \cdot \text{jn} \cdot [\alpha 1 \cdot 2, 1]$$

$$p = \text{not} \cdot \text{null} \cdot 1 \cdot 2$$

$$f_1 = (\text{insert and}) \cdot \alpha\text{null} \cdot 2 \longrightarrow 1 ; \Omega$$

example of use

let $x = \langle\langle 1, 2\rangle, \langle 3, 4\rangle\rangle$

then let $x_1 = f_3 : x = \langle \emptyset, \langle \text{rotr}:\langle 1,2\rangle, \text{rotr}:\langle 3,4\rangle\rangle\rangle = \langle \emptyset, \langle\langle 2,1\rangle, \langle 4 3\rangle\rangle\rangle$

$p : x_1 = T$. Hence $(\text{while } p\ f_2) : x_1 = (\text{while } p\ f_2) : f_2 : x_1$

let $x_2 = f_2 : x_1 = \langle r : x_1 , \langle\langle 1\rangle, \langle 3\rangle\rangle\rangle$
$= \langle \text{jn} : \langle\langle 2, 4\rangle, \emptyset\rangle, \langle\langle 1\rangle, \langle 3\rangle\rangle\rangle$
$= \langle\langle\langle 2, 4\rangle\rangle, \langle\langle 1\rangle, \langle 3\rangle\rangle\rangle$

$p : x_2 = T$

let $x_3 = f_2 : x_2 = \langle \text{jn} : \langle\langle 1, 3\rangle, \langle\langle 2, 4\rangle\rangle\rangle, \langle \emptyset, \emptyset\rangle\rangle$
$= \langle\langle\langle 1, 3\rangle, \langle 2, 4\rangle\rangle, \langle \emptyset, \emptyset\rangle\rangle$

$p : x_3 = F$

$f_1 : x_3 = 1 : x_3 = \langle\langle 1, 3\rangle, \langle 2, 4\rangle\rangle$

(3) A recursive definition of transpose

$$\text{transpose} \overset{df}{=} p \rightarrow f; g$$

where

$$p = (\text{insert and}) \cdot \alpha\text{null}$$
$$f = (\text{constant } \emptyset)$$
$$g = \text{jn} \cdot [\alpha 1, \text{ transpose} \cdot \alpha\text{tl}]$$

description

let $x = \langle x_1, \ldots, x_n \rangle$ $\quad n \geq 1$ $\quad$ and $\text{ln}: x_i = m \geq 0$ for $i = 1, \ldots, n$

(A) then for $m = 0$,

$$p : x = T$$

hence

$$\text{transpose} : x = f : x = \emptyset$$

(B) for $m > 0$, $\quad p : x = F$

hence

$$\text{transpose} : x = g : x$$
$$= \text{jn} : \langle \langle 1 : x_1, \ldots, 1 : x_n \rangle, \text{transpose} : \langle \text{tl} : x_1, \ldots, \text{tl} : x_n \rangle \rangle$$

thus for $x = \langle \langle 1\ 2\ 3 \rangle \langle 4\ 5\ 6 \rangle \rangle$

$$g : x = \text{jn} : \langle \langle 1, 4 \rangle, \text{transpose} : \langle \langle 2\ 3 \rangle \langle 5\ 6 \rangle \rangle \rangle$$
$$= \text{jn} : \langle \langle 1, 4 \rangle, \langle \langle 2\ 5 \rangle \langle 3\ 6 \rangle \rangle \rangle$$
$$= \langle \langle 1, 4 \rangle, \langle 2, 5 \rangle, \langle 3, 6 \rangle \rangle$$