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Class notes – Red languages

Syntax

We begin with a set A of *atoms*. We shall generally assume that A is composed of sequences of capital letters ~~and~~/or digits. In particular we assume:

$$\S, \emptyset, T, F \in A \quad \text{and} \quad \bot \notin A$$

$\emptyset$ is an atom which is used in place of the empty sequence. T and F are used for "true" and "false". $\bot$, "bottom", will be used later on to denote the value of a non-terminating calculation. $\S$, "unspecified" is an atom ~~expression~~ used as the value of an expression when no sensible value exists.

Formation rules for the set E of *expressions*:

(1) Any atom is an expression.

~~(2) $\S$ is an expression.~~

(2) If $e_1, \ldots, e_n$ are expressions then

$$(e_1, \ldots, e_n)$$

is an expression for $n = 1, 2, \ldots$ and

$$\langle e_1, e_2 \rangle$$

is an expression.

(3) There are no expressions except those which can be obtained by a finite number of applications of (1) & (2) ~~and (3)~~

Expressions of the form $(e_1, \ldots, e_n)$ are called <u>sequences</u> [of <u>length</u> n]. Sequences of length 2 are called <u>pairs</u>. Expressions $\langle e_1, e_2 \rangle$ are called <u>applications</u>; e_1 is the <u>operator</u> and e_2 is the <u>operand</u>. ~~ϕ is the unspecified expression~~

If $e = \langle e_1, e_2 \rangle$ or $e = (e_1, \ldots, e_n)$, then $e_1, \ldots, e_n$ are the <u>components</u> of e. e_1 is a <u>subexpression</u> of e iff: (a) $e = e_1$ or (b) e_1 is a component of e or (c) e_1 is a subexpression of a component of e.

The set $C \subseteq E$ is the set of expressions which have no applications as subexpressions; such expressions are called <u>constants</u>.

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<u>Examples of constants</u>

$$A;\ A2B;\ (X, (Y, Z, W));\ \phi;\ \emptyset$$

<u>Examples of expressions</u>

$$A;\ (A, \langle B, C \rangle);\ (\langle\langle \phi, A \rangle, (B, C) \rangle, D, E)$$

Semantics

The semantics of a Red language is determined by its <u>semantic function</u> μ, a partial function which maps an expression e into its <u>meaning</u>, μe, a constant [for some expressions e, μe is not defined]. Finding the meaning of an expression e merely involves replacing each innermost application ~~$\langle e, f\rangle$~~ $\langle f, g\rangle$ in e by <u>its</u> meaning $\mu\langle f, g\rangle$. If $\langle f, g\rangle$ is an innermost application, then f and g are constants and ~~the~~ $\mu\langle f, g\rangle = \mu h$ where h is the expression resulting from applying the function "represented" by f to g. When all applications have been eliminated from e by repetitions of the above process, the resulting constant is μe. If the process is non-terminating, μe is undefined.

Below we define μ recursively for all expressions. The definition depends on two auxilliary functions, $\hat{\rho}$ and Ψ. $\hat{\rho}$ is the "representation function" of the Red language restricted to atoms: $\hat{\rho} a$, for any atom a, is a function from C into E.

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Ψ is the "meta predicate", Ψc is true when c is a "meta expression" and false when c is a "regular" expression. Ψ depends on $\hat{\Psi}$ [Ψ restricted to atoms]. The specification of the set A of atoms and of the functions $\hat{\rho}$ and $\hat{\Psi}$ on A determines a particular Red language. All other aspects of Red languages are fixed by the following definitions:

<u>For all $x \in E$:</u>

$\mu x \equiv$

(1b) $x \in A \rightarrow x$

(1c) $x = (e_1, \ldots, e_n) \rightarrow (\mu e_1, \ldots, \mu e_n)$

(1d) $x = \langle e, f \rangle$ & $e, f \in C \rightarrow$

(1d2) $e \in A \rightarrow \mu \cdot [\hat{\rho} e] f$

(1d3) $e = (c_1, \ldots, c_n)$ & $\Psi c_1 \rightarrow \mu \langle c_1, (e, f) \rangle$

(1d4) $e = (c_1) \rightarrow \mu \langle c_1, f \rangle$

(1d5) $e = (c_1, \ldots, c_n) \rightarrow \mu \langle c_1, \langle (c_2 \ldots c_n), f \rangle \rangle$

(1e) $x = \langle e, f \rangle \rightarrow \mu \langle \mu e, \mu f \rangle$

(2) <u>For all $c \in C$:</u>

(2a) $\Psi c \equiv c \in A \rightarrow \hat{\Psi} c$

(2b) $c = (c_1, \ldots, c_n)$ & $c_1 = \emptyset \rightarrow T$

(2c) $T \rightarrow F$

To illustrate the use of these definitions we first define $\hat{\rho}$ and $\hat{\psi}$ for two atoms, DBL and FST:

result of "else" clause

$$[\hat{\rho}\,\mathrm{DBL}]\,x \equiv x = \S \to \S;\ (x, x)$$

$$\hat{\psi}\,\mathrm{DBL} \equiv F$$

$$[\hat{\rho}\,\mathrm{FST}]\,x \equiv x = ((c,d),(e,f)) \to (\langle d,e\rangle, f);\ \S$$

$$\hat{\psi}\,\mathrm{FST} \equiv T$$

We now use these definitions to find the meaning of the following two expressions.

$$\begin{aligned}
\mu\langle(\mathrm{DBL},\mathrm{DBL}), A\rangle &\equiv \mu\langle \mathrm{DBL}, \langle(\mathrm{DBL}), A\rangle\rangle && \text{by (1d5)}\\
&\equiv \mu\langle \mu\,\mathrm{DBL}, \mu\langle(\mathrm{DBL}), A\rangle\rangle && \text{by (1e)}\\
&\equiv \mu\langle \mathrm{DBL}, \mu\langle \mathrm{DBL}, A\rangle\rangle && \text{by (1b) and (1d4)}\\
&\equiv \mu\langle \mathrm{DBL}, \mu(A, A)\rangle && \text{by (1d2)}\\
&\equiv \mu\langle \mathrm{DBL}, (\mu A, \mu A)\rangle && \text{by (1c)}\\
&\equiv \mu\langle \mathrm{DBL}, (A, A)\rangle && \text{by (1b)}\\
&\equiv \mu((A,A),(A,A)) && \text{by (1d2)}\\
&\equiv (\mu(A,A), \mu(A,A)) \equiv ((\mu A, \mu A), (\mu A, \mu A))\\
&\equiv ((A,A),(A,A)) && \text{by (1c) and (1b)}
\end{aligned}$$

A lot of trivial steps can be eliminated if we observe that μe can be found by repeatedly replacing an innermost application by the appropriate right hand side of (1d2) through (1d5) but without the "μ".

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Thus the same evaluation looks like this:

$$
\begin{aligned}
\langle (DBL, DBL), A \rangle &\xrightarrow{id5} \langle DBL, \langle (DBL)\, A \rangle \rangle \\
&\xrightarrow{id4} \langle DBL, \langle DBL, A \rangle \rangle \\
&\xrightarrow{id2} \langle DBL, (A\ A) \rangle \\
&\xrightarrow{id2} ((A\ A), (A, A))
\end{aligned}
$$

We now use this latter method to evaluate our second example which involves the "meta expression" FST.

$$
\begin{aligned}
&\langle (FST, DBL), (A, B) \rangle \\
&\qquad \xrightarrow{id3} \langle FST, ((FST, DBL), (A, B)) \rangle \\
&\qquad \xrightarrow{id2} (\langle DBL, A \rangle, B) \\
&\qquad \xrightarrow{id2} ((A, A), B)
\end{aligned}
$$